Physics Lecture 7 - The Center of Mass (CM) Effect

Summary/Conclusions

For a track with a circular arc ramp, there is a significant difference in finish time/position depending on the CM position. For a track with an inclined plane ramp there is only about 1/5 (23%) this effect. For a track that is all inclined plane there is zero CM effect. It takes some pretty detailed calculations and applied physics to figure this out but we will spare that detail here.

Introduction

This material, in summarized form, is taken from the Physics of the Pinewood Derby book which contains the relevant equations of motion and more detailed solutions and results for the following CM cases. This lecture will strive to explain this effect with only a little math that even a 5th grade middle school child can understand. Almost everybody believes, correctly so, that the higher the CM of an object the more potential energy it has and the greater velocity will be developed as it falls a certain distance, say *h*. And, for a rigid body, we can suppose all its mass is concentrated at a point called the CM. The velocity a body will receive falling <u>vertically</u> a distance *h* subject to a gravitational acceleration *g*, where air resistance may be neglected, does not depend on mass and is given by $\sqrt{2gh} = \sqrt{2g}\sqrt{h}$. Now *g* for the earth is 32 ft per sec^2 and if we drop something from a height *h* = 4 ft and recall $\sqrt{64} = 8$ and $\sqrt{4} = 2$ we see it will strike the ground at 8 x 2 = 16 ft/sec which turns out to be 11 mph. Now if we slide the body down an inclined plane or whatever ramp shape so that eventually it still drops 4 feet we still get 11 mph at the end (with no friction) although it will take longer to build up to this speed. Now, that's it for the math—not too bad, huh? So now we are ready to consider the 3 tracks of **Figure 1.** Study the Figure closely.

The Circular Arc Ramp

As it turns out, if you follow the directions for a pinewood derby ramp from the 1991 printing of the Cub Scouts *How To* book, on p 40 Chapter 9, you will have a ½" thick plywood ramp that will naturally sag so that it approaches a circular arc. The Piantidosi aluminum track also approximates a circular arc ramp. Now focus on the top view of the circular arc ramped track of **Figure 1**. We have 3 identical cars A, B, and C with their CMs close to the front, center, and rear respectively. Let the CM position at the center be zero for car B, then the CM at the front for car A is about - 3" (-7.62 cm) and that for car C is close to the rear axle for + 3" (+7.62 cm). If your wheel axles are in different spots, then that may restrict your CM positions more than the wheelbase used in **Figure 1**. If the CM isn't between the axles your car may do a "wheelie".

Suppose you take 3 snapshots, one just before the start gate opens, a second when the slowest car's CM just hits the end of the ramp where coasting starts, and a third when car B's front bumper triggers the timer stop signal. Now a car rolling down a circular arc behaves just like a pendulum in a grandfather's clock or like pulling a yo-yo back a certain angle and letting it swing at constant string length in a circular arc. It turns out, and you can try this, that the time for the pendulum mass to reach the horizontal does not perceptibly depend on the height at which you release it (as long as you don't raise it *way* high). Look at the side view of ramp 1. Car C with its CM at the rear axle (+ 3") gets to start off with its CM falling closer to vertical (sharper angle at curve tangent where CM is) than car B's CM so the fraction of gravity's force accelerating its CM is greater and by the time the horizontal is reached it just barely catches up with car B's CM which had a head start. The same can be said of car B's CM relative to car A.

Now here's one advantage that favors car C over car B over car A. Even though their CMs all hit the horizontal at the same time (there is only a tiny, <u>tiny</u> difference) C is moving faster than B which is moving faster than A according to the *h* of their CM plugged into $v = \sqrt{2gh}$. So if you are sitting in the middle car B at the ramp bottom (point P) and you look directly left you see car C's driver sitting on his CM grinning from ear to ear because he knows he is going faster than you. Not only that, he knows he has all that pinewood sticking out in front of him to poke the finish trigger and you have only half as much sticking out in front of you. And you look to the right seeking some consolation and you see car A's driver sitting on his CM at the front of the car with a really big frown on his face mumbling something about his blankety-blank dad making him put all the lead at the car front. He knows that not only is he the slowest entering into the coasting part but he has almost no wood to reach out and hit the finish trigger. So there are 2 effects of the CM position—the velocity effect and the front end extension effect. If you do the math it turns out they are of comparable importance time-wise. The tracks here are 16 ft ramp and 16 - 3 = 13 ft coast with *h* = 47" as measured vertically from start (at CM=0 point) to finish track surfaces. The actual start posts are 7" from the ramp top and the 3 ft is stopping distance. **Table 1** shows car C can beat car A by 1.5 car lengths at the finish line on this circular arc ramped track.

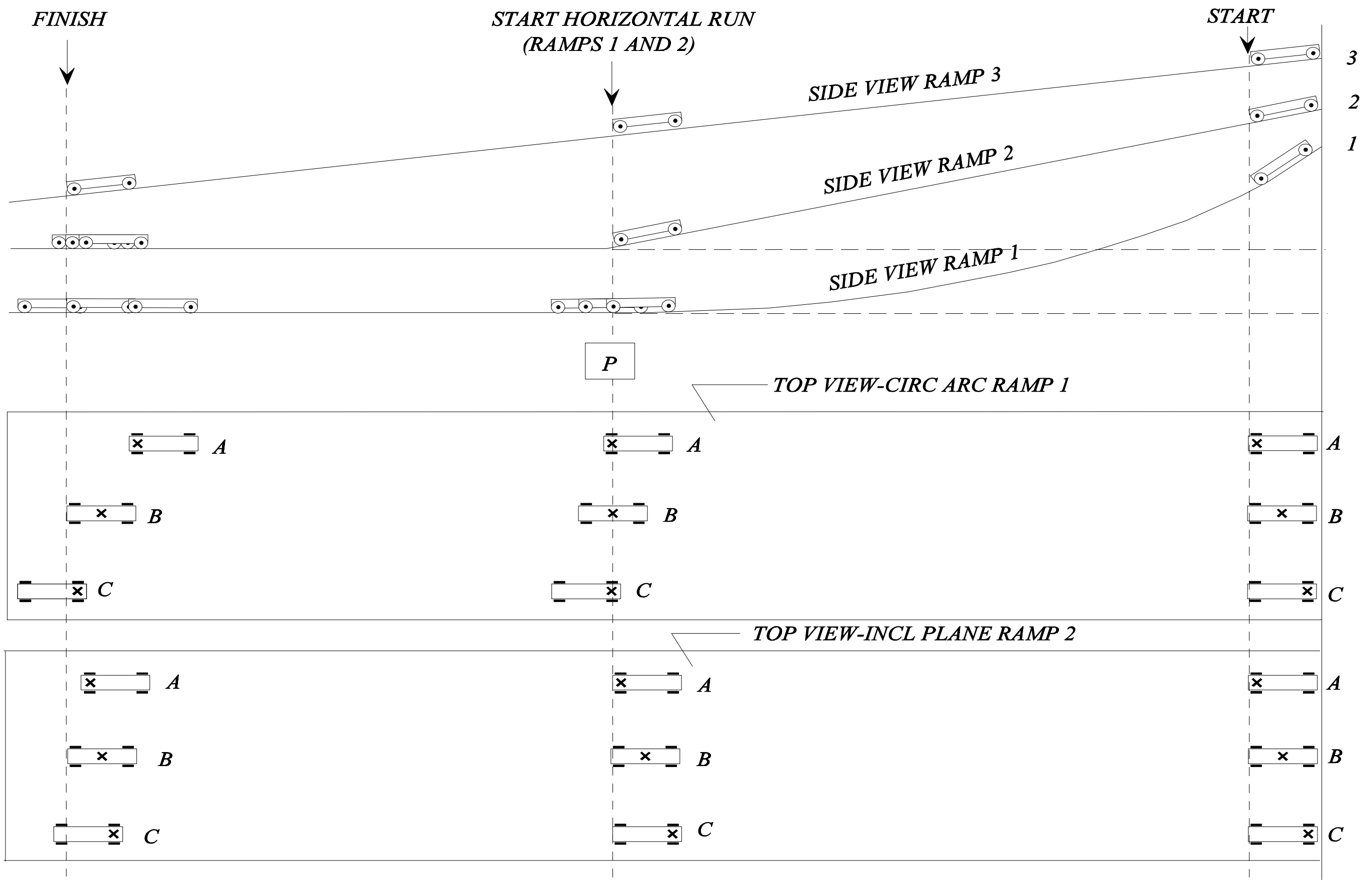

Figure 1 - Track 1 has a circular arc ramp, track 2 has an inclined plane ramp, and track 3 is all inclined plane.

The Inclined Plane Ramp

Next consider cars A, B, and C on a track with an inclined plane ramp as in **Figure 2**. The BestTrack is an example of a commercial aluminum inclined plane ramp with only a slight transition curve at the ramp bottom. Each CM has precisely the same component of gravitational force accelerating it because all are constrained to move at the same angle (it turns out that this angle is exactly ½ the starting angle of the circular arc ramp with the same *h*). If the distance down the ramp is 6*h*, then the common acceleration of all is 1/6 *g*. Thus the CMs under the same acceleration will travel equal distances in equal times and so all the bumpers will be lined up as car B's bumper hits the end of the ramp. Now car B's driver, sitting on his car's CM, looks to the left and there is car C's driver and CM about a half car length back. He has sort of a grin because he knows car B has only a half car length of acceleration left while he still has almost a full car length of the same acceleration left. When car B's driver looks to the right, he sees car A's driver almost at the ramp end with not much acceleration distance left. It turns out that car C's higher velocity at the end of the ramp is just enough to barely overtake the head start (but eventual lower velocity) of car B, and car B will beat out the unfortunate car A by this same amount. Note that the all inclined plane track 3 has no CM effect. The VR gives **Table 1**.

Table 1. *Race times for the circular arc and inclined plane ramped tracks. Units are cgs (centimeter-gram-second)*

Ramp	Car	CM *(cm)*	*ramp t =T1(s)*	*ramp v = V1 (cm/s)*	*coast t = T2 (s)*	total *t = TT (s)*	Δ*TT (s)*	Δ car lengths at finish
circ arc	A	-7.62	1.5559	475.585	0.8305	2.3864		
							0.0279	0.754
circ arc	B	0.00	1.5568	483.152	0.8017	2.3585		
							-0.0269	-0.741
circ arc	C	7.62	1.5576	490.710	0.7738	2.3314		
incl plane	A	-7.62	1.9355	479.623	0.8235	2.7590		
							0.0066	0.179
incl plane	B	0.00	1.9513	483.544	0.8011	2.7524		
							-0.0063	-0.173
incl plane	C	7.62	1.9670	487.433	0.7791	2.7461		